SUMMER 2017 ISSUE NO. 4

ROM

A RECORD OF MATHEMATICS

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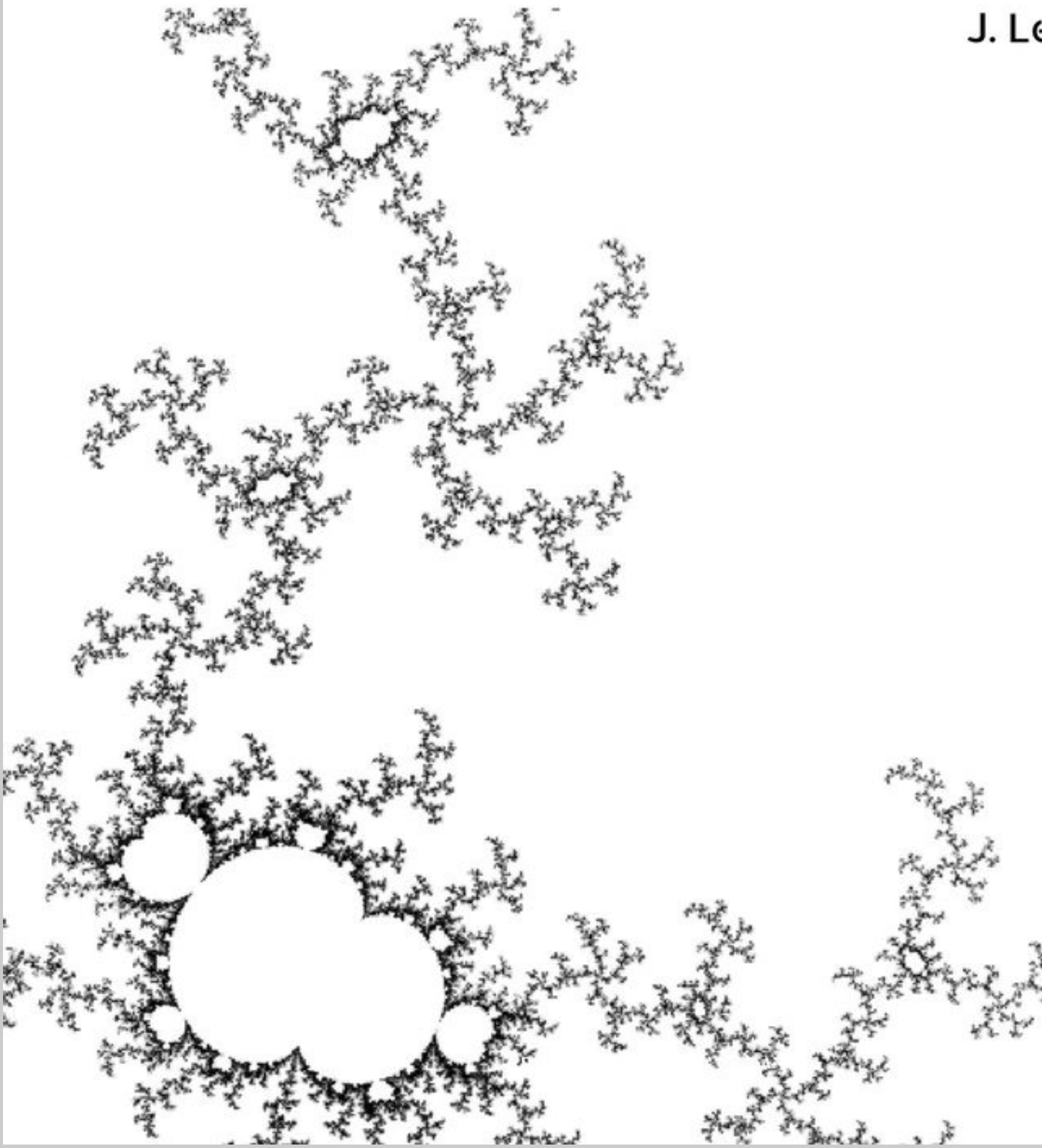


What Font is This

As a continuation of the tradition based on the previous issues, we will introduce this current font, "Crimson Text". Note that the font may change occasionally for specific purposes, and the new fonts may not be introduced. But please be assured that this font will be the most commonly used and shall be used whenever possible. You may never discover the significance of this on your own, but we are not about to tell you. Good luck.

Schedule

• Jonah & Lucas's Branch class is in Walton 21. Where's that? Enter Smullin the usual way, then pretend you're going to the bathroom, but turn *left* at the second fork instead of right towards the bathroom. Follow the signs towards Walton 21, which is down some stairs.

• Friday schedule:
8:05 AM — 8:55 AM: Breakfast
9:00 AM — 12:00 PM: Morning class
12:00 PM — 1:00 PM: Hey, let's clean up our classrooms.
1:00 PM — 1:30 PM: Lunch
1:30 PM — 5:00 PM: Cleaning, packing, and exit interviews in the dorm.
5:00 PM — 6:00 PM: Closing meeting
6:00 PM — 6:45 PM: Dinner

• Saturday schedule:
Go home.

• Daily Gathers: Noah Forman, Robert Lipshitz, and a Willamette REU.

Stop Reading and Get Back to Playing Games

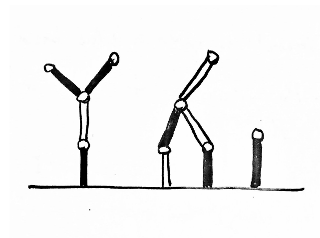
Julia

Wooooooo! Jonah's branch class! Filled with games! And fun! And… Godzilla?

Our main focus this whole week has been centered around really getting down to the very core of games like trinkets/zugzwang, pebble parades/square and rectangle game, and lots of other new ones that share some similar attributes. Here are just a few of the games we've been scouring, as well as some of the really cool things we discovered about them!

Deforestation:

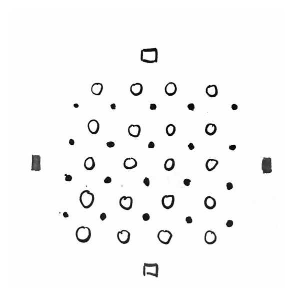
Deforestation is a game that involves a bunch of tree graphs that are connected to the ground. Each edge, or branch as we call them, is colored either black or white. The game consists of two players each taking turns removing a branch of *their color*, meaning that if you're removing white branches, you can't remove any black branches. Here's what one particular board might look like:



The fun thing about deforestation boards is that the order that the color of the player who goes first might not always matter, because in some boards, the white player will always win, or in others, the black player will always win. This concept brought us to the idea of placing value on different branches so that the total value of the board could indicate from the start who would win. The black branches have positive values and the white branches have negative values. When the total value of a board is added up, black will win if it's positive, and white will win if its negative. And if the value is zero, then the second player wins, regardless of color.

Fiberjourney:

Named in the spirit of Word Assassin, fiberjourney is a game made up of a grid of white dots and black dots. They are arranged so that the white dots create their own grid, and the black dots create their own grid that's centered between the white dots, like so:

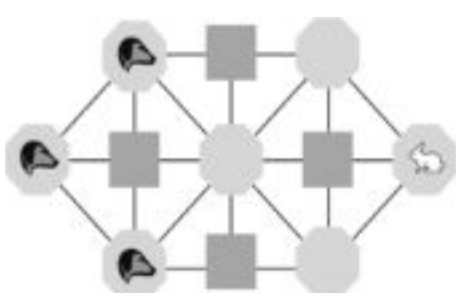


In order to win, a player must make a path from one end of the board to the other by making connections between their color of dots. The black player must navigate from side to side, or the white player from bottom to top. We found that there was a good strategic move, which we called a Double Attack, where a player could set up two possible moves that will achieve their goal, so that if their opponent tries to block one of them, they can use the other. We also conjectured that it is impossible to have a draw in this game, as in order for one player to lose, their potential pathways must all be walled off. And walling off one players pathways means that the other player has reached both of their ends.

We also found that this game is possibly an exact copy of another game that we talked about, which we typically referred to as Godzilla games. In this game, you're given a map of streets, much like a graph, and you, the Department of Public Works, and your opponent, Godzilla (duh), take turns either destroying or fortifying streets. The DPW's goals is to fortify a path of streets to get from a predetermined point A to point B, where Godzilla's goal is to block them by destroying streets that have not yet been fortified.

Dogs n Bunnies;

On Saturday at the end of class, we were introduced to a new game which we currently are still struggling to find strategies and conjectures for. This one takes place on a board of square and octagonal cells, as shown below, with three hounds on one end of the board, and a single hare at the other end. One player plays as the dogs, who can only move forwards or sideways and one at a time, and the other plays as the hare, who can move in all directions. They take turns moving their pieces, and the goal is either for the dogs to trap the hare in a cell where it can't move anymore, or for the hare to escape the dogs advances.



We currently have many different conjectures about this game; Many claim that the dogs can always win, regardless of board size or who goes first. Others claim that the hare can always win, and other still suggest that it all depends on who goes first. We still have a lot to find out about this game in the coming week, but so far it looks like something we've never seen before!

So yeah, that's just a brief summary of *some* of the things we've been doing in Jonah's Branch Class. We've learned about plenty of other games as well, but a lot of them share extremely similar concepts with the ones above. It's been really fun so far -- I mean, like, we basically just get to play games for seven hours a day -- but along with all the fun is some really intense, cool, and mind-boggling math stuff. It's pretty sick, my dudes. Bet ya wish you were there.

Non-Euclidean Geometries

Patrick

Topics of interest in Alice's non-Euclidean geometry class included geometry on the surface of a sphere, geometry in the plane of complex numbers, and various geometries on the projective plane.

At the beginning of the week, the class spent several hours redefining basic concepts such as lines and regions which lose intuitive meaning in other geometries. In spherical geometry, a line is not straight; it is a circle which circumvents the surface of a ball. In projective space, lines aren't parallel in the traditional sense. They converge at some "point at infinity," the same way parallel lines appear to converge at a vanishing point in the distance. Projective geometry is best compared to a floating eyeball on the plane, looking directly at objects around him. Lines pointing towards the eyeball appear as points, and parallel lines disappear into the distance and appear to be a single, continuous figure.

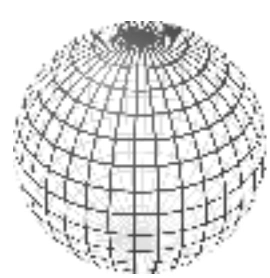


Figure 1: A Sphere

Most people know that the real numbers can be expressed on a “number line,” and many know that the complex (imaginary) numbers can be expressed on the “complex plane.” In this plane, imaginary numbers represent the y-axis and reals represent the x-axis. We can do geometry on this plane! We used functions to describe lines in the complex plane, and showed we can invert them into circles near the origin. These inversions yielded some interesting observations, and it is now conjectured that it may not actually be a “number plane,” but rather a sphere, or even a torus.

We extended the complex plane even further. We tried to do geometry using three dimensional numbers, but this proved difficult as these numbers did not have some of the properties we needed. So we extended this into four dimensions, into a set of numbers we call the “very simple numbers.” The very simple numbers feel very close to previously studied topics in algebra and number theory, and we look forward to exploring their properties next week.

Untitled Colorings

Adrian

This Monday, we worked on choice coloring in the daily gather, one of the many topics studied by Maryam Mirzakhani. Choice coloring works like this: We see a graph and choose a number, *k*. Our worst enemy chooses k colors for each vertex, which we can choose from to color each vertex. We want to choose the smallest k such that we can still color the graph. We saw that choice coloring is different than normal coloring. We discovered that we can make charts need more choices than they do colors. As shown by our charts, it's possible to make a map that needs five colors and a two colorable graph that needs 3 choices. This is strange, really cool, and really really weird. As Jonah showed, our classmates can be wrong, and, not only that, but very wrong. That was kind of sad but an important lesson. Also, we learned a lot about how weird rules can change stuff and first thoughts can be deceiving. Mirzakhani was a genius, solving this all without hints at age 19.

Bouncing Balls on Billiard Boards

Andrew

On Tuesday Alice presented us with a square billiards board. We would choose a starting point for our ball and shoot it in a direction where it would bounce off the walls endlessly. Our task was to find out things about this board, such as will the ball ever return to its starting position or will patterns emerge in the ball’s path? At first most people just drew such boards and paths to familiarize themselves with the problem. The main patterns that people noticed were that the paths almost always devolved into either forming several rhombuses going in a single direction or forming a rectangle at a 45° angle. Most of the daily gather was spent trying to figure out when, if ever, will the ball return to its original position?

Most people’s intuition led them to the slope of the path that the ball followed. Some speculated that when said slope is rational, the ball will return, but when it’s irrational it will continue endlessly without ever revisiting a point. Some people tried to prove this outright without modifying the board or parameters at all, but came up short.

One idea that Adrian came up with was to duplicate our original square board, but reflected over the top line. Continuing this over the right side of our now 2x1 board we can get a 2x2 board of our original squares. With this we were able to treat the board as if it were continuous, such that when it ‘hit’ the right side it came out the left instead of bouncing, or vice versa By doing this it simplifies the problem so that our slope is constant and never changing when it bounces off walls. I don’t think we ever officially proved that rational slopes return to their original points, but at the end of class Alice told us that our intuition was right; rational slopes will return to their original slopes while irrational slopes will continue on without ever revisiting a point they had already been to.

Even More ꟼirafs

Lexa

By the second to last week of the program, we still had not made *all* the graphs. To fix this, Willamette REU researchers showed us a new way to think about graphs: intersection graphs.

The first kind of intersection graphs we looked at was interval graphs. These graphs are generated from sets of parallel labeled line segments, as shown below. Each line segment corresponds to a node, and the node is connected to any other nodes whose line segments "overlap." Thus the example below generates a three-node necklace.

A ---------------

B ----------------

C -------------------------------

Then, we looked graphs made from intersecting areas instead of lines. Such intersection graphs worked similarly, with a set of labeled areas representing a graph whose nodes corresponded to areas and connections corresponded to intersections between areas. In particular, the researchers were interested in intersection graphs made by finding the maximal-sized convex shapes in a given polygon, and using those as areas. The catch: each polygon and its set of shapes was restricted to a certain set of angles. To describe these objects, the researchers used the following terminology:

- A **k-lizard** was a polygon with k angles allowed, and no holes. E.g., a rectangle would be a 2-lizard and a hexagon a 3-lizard.
- A **scale** was a maximal-sized convex k-angled subpolygon of a k-lizard
- A **k-MSP** graph was a graph generated from the intersections of all scales in a k-lizard

Thus armed with knowledge we set out to find k-lizards if possible for different kind of graphs, such as full graphs, cycles, and "caterpillar" graphs. We investigated whether such graphs could be k-MSP's and if so, for what k's.

So Many Questions...

Rocio

On Thursday we had fun playing the 20 questions game!

Have you ever heard of this game? If not, I'll explain it to you in a few sentences. We had 20 Yes/No questions and we could guess a number between 1 and 2^{20} if all the answers were true.

We were challenged to find the maximum interval of numbers we could guess with certainty if there was just one lie among the answers. In the beginning, we thought that the best answer would be 2^9, but surprisingly, Lexa and Yuyuan came up with a great method which could help us to guess a minimum of 2^{14} values and a maximum of 2^{18} in the best case. Their idea was asking 14 questions, and in the 15th, ask if they had already lied. This is not necessarily the optimal solution, but this was the best that we came up with. It was a great daily gather and their method was definitely useful and cool. Well done!

End of the Knot (or Not Knot?) Odyssey

Oskar

Friday's Daily Gather was held by Hannah, MathILy-Er instructor and avid fan of 3-manifolds, so I think you can guess what it was about. Of course, knots!

We first received a sheet with different knots and notations and we had to determine what the notation refers to. It turns out that it is simply the number of crossings that the knot has. Then Hannah showed us how to create graphs from knots by transforming intersections into vertices. These graphs had the following properties:

- all graphs are Lien (clean) since all intersections become nodes
- no graph is humble
- for graphs with n nodes, there are n+1 spaces and 2n edges

Next, we tried to two color the knots with red and white so that no space is adjacent to a space having the same color. From these colored knots we created medial graphs by putting a vertex into each space and connecting it to its adjacent spaces through the vertices. To be able to recreate the knots from these graphs we put + or – next to the edges so that the + indicates that the edge passes through an intersection where, if we hold the knot so that the two spaces above and below it are red, the right part of the string passes under and the opposite when we have a -.

For the last few moments, we thought about how the medial graph would change if we applied Reidemeister moves (pretty fancy name, right? (Answers to this question can be sent to my mailbox)).

Several knots and their notations