Herding Cats Using Purrology

By: Alessandro

Annie explained that all throughout MathILy-Er she, and other MathILy-Er participants, have been having trouble controlling cats during class! Hence, it is a matter of absolute necessity to find a solution…
By herding cats!

We started out with some feline facts:

A purrology P on a collection of cats C is a list of happy herds that satisfy the following three rules:

Three Rules for Happily Herding Cats:

(1) The herd of "no cats" and the herd of "all the cats" are both happy.
(2) All unions of happy herds are also happy.
(3) All intersections of happy herds are also happy.

(note: a "herd" is just a bunch of cats who are all from from C, our original bunch)

Presumably, putting cats in happy herds would prevent them from bothering their owners during class, hence, it would be beneficial to learn more about the strange ways of the cats.
In order to accomplish said learning, the class addressed the following questions:

How many possible purrologies exist for 3 cats?
How can we order different purrologies?
What would make purrologies chonkier or smoller than another?

Through casework we learned that there are 29 different purroligies for 3 cats. It was also revealed that the different purrologies could be ordered into 9 distinct groups. Finally, it was decided that for herds of cats A and B, A is **chonkier** than B when A has more elements than B, and B is contained in A; A is **smoller** than B when A has fewer elements than B, and A is contained in B. It directly follows that the purrologyof the sets of all cats and no cats, i.e {abc, Ø}, would be the smollest purrology, and the purrology with every possible union and intersection, {abc, ab, ac, bc, a, b, c, Ø}, would be the chonkiest purrology.

We then moved on to MORE CATS! How many purrologies are there for more than three cats?

Here it was conjectured that there are n^n+ (n-1) purrologies for n cats. This was shown to be true for 1, 2, and 3 cats… but it remains unknown if it holds for 4 cats and beyond as the time for Daily Gather had run out.

Annie ended Daily Gather by revealing that the herding of cats was really showing us some of the basic definitions in point set topology using finite sets. In essence, those were some seriously mysterious cats!

Stage Directions, Towers of Hanoi, and the Dilemma of an Ant

By: Maanas

Corrine wants our help to write her next play! She is inspired by this structure of us having every subset of the four actors appear on stage exactly once. And, between any two consecutive scenes, only one actor may move: either on stage or off stage. Can she do this? If so, let's write it nicely!

We decided on making a table for the scenes in order, where a 1 notes an actor being present in the scene and a 0 represents being absent! Then, we found a process for making these tables based on previous tables with one fewer actor!

To get the stage directions for n actors: take the stage directions for n-1 actors, then take another copy of the n-1 directions and write them after the first copy in reverse order. For the first copy, put 0's in the nth column, and 1's for the second copy! Ta-Da! Though Corrine loved this idea of having a mystery character appear halfway through, Heather McNamara, a 100% real attendee of MathILy-Er 2020, was a big fan of halving things in our solution. (Note to self to check in on her.)

Corrine introduced the Towers of Hanoi problem, and the dilemma of an ant trying to reach a corner of a cube exactly once. The solutions to both of these problems are somehow related to our stage directions, but how?! I guess you'll just have to wait for it!

THEATER PUNS.

Why Can't We Have Nice Things?: Wherein Our Shiny Illusions of Democracy are Destroyed

By: Maanas & Amanda

You might reply, as an answer to the question posed in the title, that we do have nice things. You might say, isn't democracy a nice thing? (Hah, imagine thinking democracy exists). Nevertheless, as we shall see, there are some rather jarring negatives. But before we get to that, we'll have to travel back in time to the start of our fifth and final :(week of MathILy-Er...

New And (Not So) Improved

We began class by modifying our definition of a voting system. Now, we defined a ranked voting system as a function from the set of all possible elections to the set of all rankings of the candidates, maybe with ties (Previously, we had defined it as a function with an output of one or more winners).

As we proved a smattering of properties throughout the week, we also discovered a rather strange phenomena: certain properties could not be paired with other properties, and/or if they did, strange things would happen... For example, a voting system could not have both the queen bee property and the winnin'4'eva property. Also, a voting system with the consensus and third wheel properties paired with an election with at least three candidates results in a tie.

Lastly, we explored a new type of voting system -- one where everybody simply votes yes or no for each candidate. This voting system had more nice things than we thought it would, but also had its own drawbacks. "Don't go sleepin' on other voting systems" Jonah said.

You Get A Dictator, You Get A Dictator, Everybody Gets A Dictator

We defined a dictating group as a group of voters where, if all of them vote for A over B, then A will beat B. In other words, if they vote as a block, they can be a dictator. Specifically, an A→B dictating group: a specific dictating group in which if all members of this group vote for A over B, then A will beat B, but this doesn't necessarily apply to other candidates.

We proved that if a system has both the consensus and third wheel properties, then in any election where A is ranked over B the set of voters who voted for A over B is an A→B dictating group. This was interesting because aren't the consensus and third wheel properties supposed to be good things? But wait… it gets worse…

After quite a lot of math, we showed that if V is an A→B dictating group, it can dictate all other pairs of candidates too! And when we split V into two smaller sets, at least one is a straight up dictating group too… What!? If we have the consensus and third wheel properties and at least three candidates, then we have a dictating group? And if we have a dictating group, then there's actually a dictator? *Now* are you freaking out? Two properties that we like and seem perfectly normal actually are dictatorship voting methods in disguise! And things even worse when we look at representatives...