
MathILy-ER
2022
WEEK OF CHAØS[3]
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COVER ART: KALLIE WANG
RECORD OF MATHEMATICS

Record of Mathematics: Week 3

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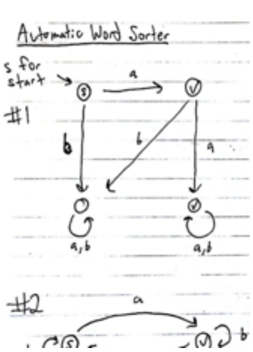
1.10 That's Numberwing by Tom (Janice)

We started the class by considering 2 jars, one of which measures 5 gallons, while the other measures 3 gallons. Our goal is to figure out a process to use those two jars and accurately measure out 1 gallon. We realized we can fill up the 3 gallon jar and pour the 3 gallon of water into the 5 gallon jar. Then, we fill the 3 gallon jar again and pour into the 5 gallon jar and fill it up. This leaves only 1 gallon of water remaining in the 3 gallon jar. We extended this question by considering what measurement in gallons can you get out of using 2 jars, each with a measurement of m, and n gallons. So we can translate this in the equation mx+yn = c, which m, and n are the measurements of the two jars, x and y are integers, and c is the integer gallon measurement we can possibly get. There are two scenarios. First is when m, n, are coprimes, then we can always have some integer solution x and y for any integer c that satisfies the equation. The second scenario is when m,n are not coprime, their greatest common divisor has to also be a divisor of c for the equation to be solvable for some integer value x and y. After proving this, we wondered if there is the same idea of coprime, or prime factorization in complex numbers and what are their characteristics?

1.11 Erdős ˆMagic by Corrine (Jason)

Erdős ˆMagic involves the clever utilization of probability and expected value to prove many combinatorial questions, specifically of the existence of things. After we shared our background knowledge of probability and expected value, Corrine guided us through many concepts such as random variable functions and sample spaces. We also learned about linearity of expectation, written as $E[x + y] = E[x] + E[y]$ for events x and y, which would prove vital for many more problems for the rest of the week. Of course, root class knowledge came in handy for many of our proofs. We delved into a lot of detail on the probabilities and expected values of various properties of districts/states. For example, we spent part of day 4 and all of day 5 on the following setup:
Say we have a set of n buildings/towns. For each pair of towns, independently flip a coin that has probability of heads as p to decide whether there's a road between the two towns.
We formulated many questions with this setup, such as the expected number of loops of length k. Some strategies we learned during our magical explorations were to focus on the bad cases and to fix certain elements of the problem before moving to the general case. For the question detailed above, we first looked at a specific loop of length k before moving to the general case. Of course, this was just one of the many problems that we looked at throughout the class, all of which involved the magical concepts from probabilities and expected value.

1.12 Wind Up Birds by Kimball (Kallie)



In Wind Up Birds, we explored Automatic Word Sorters(AWS). Circles represented "states". Arrows were used to move between states, depending on if your inserted word has an "a" and/or "b". If your word ends in a state with a check mark, it is considered a "happy word". For example, in AWS 1 to the left, by following the arrows one letter at a time, aab is a happy word but abb is not. In AWS 2, a word is happy if there is an odd number of a's because if an even number of a's are inputted, the word will not end on the check mark. Then, we explored number divisibility by 2, 3, 4, 5, 6, 7, etc. with AWS machines on Tuesday. On Wednesday, we asked if there are any problems AWS machines cannot solve, in which we conjectured the set of primes would not be able to be sorted by an AWS with finite states. We found ways to make the intersection and the union of 2 AWS machines. We expanded on Indecisive Word Sorters (IWS), Memory Automatic Word Sorters (MAWS), and infinite AWS machines that allowed for more possibilities for different types of AWS machines.

1.13 Bird Den of Proof (Jeffrey)

In this week of chaos course, Max led us through 5 different proof methods (one per day), with a potpourri of problems to solve each day. On Monday, we learned the direct proof, where we use information given and manipulate it to conclude what we want to prove. On Tuesday, we learned induction and used it to prove inequalities and sums. The next day, we learned the proof by contradiction, where you assume the proposition to be false, leading to a contradiction, implying that the proposition must be true. On Thursday, we learned the proof by construction, where we show something must be true by creating it or giving a method to create it. On Friday, we used proof by contrapositive, which states that if not Q then not P implies P then Q.