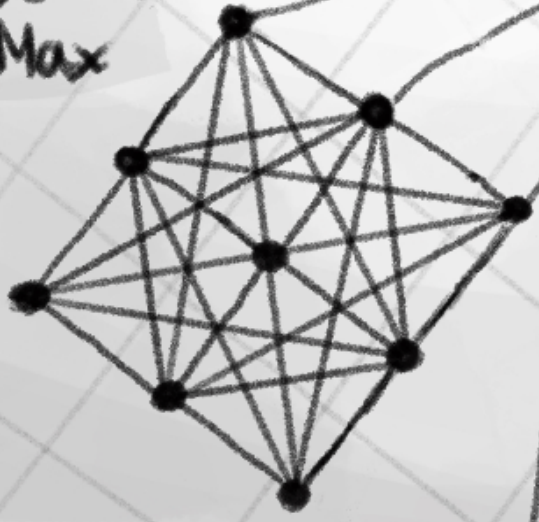
By Rohan and Ivan

teD $\begin{bmatrix} MAT \\ HIL \\ YER \end{bmatrix}$

AR θ [M]
WEEK 5

"Get bozoed"
-Max

$f(x) = \frac{ax+b}{cx+d}$



$t \mapsto t$

Record of Mathematics: Week 5

Claudia, Janice, Kallie, and Mary

August 2022

Contents

3 Daily Gathers

3.1 Monday: The Hairy Ball Theorem by Nate Harman (Martin)

Today, we were introduced to the Hairy Ball Theorem: a hairy ball cannot be combed flat. A hairy ball is a sphere with a hair at every point which is combed to be tangent to the sphere. A ball is combed flat if the directions the hairs are pointing vary smoothly, so if you were to zoom into the surface of the hairy ball, all the hairs would appear to point in roughly the same direction.

First, we found the many ways this could go wrong to get an understanding of hairy balls. We also observed a simpler version of this theorem on a circle and a donut. We found that a hairy circle and donut could be combed flat.

To try to prove this theorem, we used something we understood more concretely: unidirectional webs/states. We identified how hairy balls could go wrong in terms of these webs/states, finding ghost towns (all teleporters pointing away), fire hazards (all teleporters pointing towards), and small loops (counties and teleporters forming a loop with nothing inside). We proved that one of these must exist on a finite web. This web was then superimposed onto the hairy ball, and all the directions the hairs were pointing in were determined by the nearest teleporter direction. Since a fire hazard, ghost town, or small loop were ways the hairy ball could go wrong, it was proved that a hairy ball could not be combed flat.

3.4 Thursday: Pixie Spices by Alice Mark (Joaquin)

It's the Final Daily Gather! Alice was back, 29 days after her last daily gather, to give the final one. We started by looking at roots where every path was expressed by a string of Ls and Rs. Then, we did "something weird" to those strings and added an L to the beginning and duplicated the last character. After that, we looked at the length of "runs" of characters, added a 0 to the beginning, turned it into a fixed fraction and then finally unfixed the fraction. After looking at the fractions for every node of the roots, we saw that they looked a lot like the pixie series.

Since the Pixie Series is generated by waddition, the question was how we found the two Left Right sequences' fractions to wadd to get any Left Right sequence on the roots's fraction. Eventually, we conjectured that you can remove everything from the end of a string up to and including the most recent L to get one of the strings, and everything up to and including the most recent R to get the other string. Now we had to prove this always works.

We decided to represent the fixed fractions as spice racks to help us. A general form for the spice racks generated from two strings we are wadding together was found, and we realized there are 4 cases based on if the last two "runs" of the string have length 1 or not. When we did prove it by manipulating the spice racks to get what we wanted, everybody said "OOOOH!" and "WOOOOW!". Also, Alice is a liar since she said she would never do a Daily Gather again and they did plan things.