the MathILy-Er
Record of Mathematics (RoM)

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3.2 Tuesday: Fashionably Superstitious Worker Bees by Lingfei Tang

Imagine if one day, your brother suddenly turned to you and asked the question: "If n is my favorite number, can I always find a Fibonacci number divisible by n?" What would you say or do in this situation? Brian's response was a perfectly reasonable "I dunno, probably." But was he right?

We managed to prove that this was true by arguing that the Fibonacci numbers $mod\ n$ eventually loop because there are a finite number of consecutive pairs, but this only led to more questions. What is the first number divisible by n? How long is the loop? What about different Fibonacci-like sequences? If we can always find 0, what about other numbers $mod\ n$?

A counterexample was quickly found for the last question, while the second turned out to be more complex. Some ideas proposed were looking at the position of the first 0 and using a computer program to generate data, it was conjectured that the loop length is the lcm of the terms $loop(p)p^{a-1}$ for every p^a in the prime factorization. We considered some small cases, but this is still an open problem!

3.3 Wednesday: 53rd Annual Film Festival by Geoffrey Smith-Donald

Wednesday's Daily Gather was the 53rd annual MathILy-Er Film Festival which we started off by watching the second episode of numberwang. It was quite a close match with Simon defeating Julie after an intense round of numberbounce.

A little bit later we watched the dragon hiding in a tree which showed how the dragon curve appears in the Pythagorean tree in an unexpected way. The video also demonstrated some relationships between the tree and other fractal curves.

Another favorite of mine was the shape of space video which described some of the different shapes our universe could be while appearing the same to us. It had some nice visuals which assisted in describing how the seemingly 3d space could be connected to itself at the edges.

We finished by re-watching our favorite video from last week's film festival, "Meditation on Homotopy of Embedding" which was incredibly enlightening and received a deserved thunderous round of applause.

3.4 Thursday: Duck Goose Chicken Goose Duck by Arav Paladiya

Kye is usually at MathILy, where they have a tradition of doing a Daily Gather at the Haverford duck pond every year. The many birds (chickens, ducks, geese, and swans) who sit around the pond each want to interact only with birds of species other than their own. Birds interact with their immediate neighbors. Since you look different on your right and left, an interaction must occur in each direction. Kye asks: what is the minimum number of birds we need so that every species of bird can interact with every other bird? Many people came to the same conclusion that 12 birds are needed, with 3 of each type. Some people also came up with a way to visualize this like a city/CAMP.

After this, Kye asked us to make a configuration with all possible triplets of birds next to each other, accounting for order. Additionally, he gave us a hint to create a city where the monuments/students were the pairs of birds. We found this takes 24 birds. But when asked to present, nobody wanted to; so Kye simply left the class, only to very coolly appear again through another door.
Then Kye asked us to visualize this in a better way where the lines between the pairs of birds didn't intersect. He used sage to plot a 'digraph' of the configuration, which looked suspiciously 3-dimensional. He also plotted a shape called a cuboctahedron, so that we could see how similar it was to the digraph. Kye finished his Daily Gather with a motivating speech on remembering our humanity and trying to make the world a better place.

3.5 Friday: Flea Swatting by Alice Kim

The universe started with the big bang: an explosion of an infinite number of fleas. These fleas each corresponded to an integer in their line-shaped universe, and every second they would all leap away from the origin, with the flea corresponding to integer k jumping a distance k. Since we hate fleas, our ultimate goal is to "swat" all of them using a swatter that extinguishes fleas at one point of your choosing every second.

The question is: assuming that we have an infinite amount of time and resources, will we be able to swat all fleas eventually? With fleas corresponding to every integer, Aiden came up with a formula that ensures all integers in our line-shaped universe eventually become swatted by our swatter.

Unfortunately, there exists an alternate universe in which there is a flea for every real number. To face the seemingly uncontrollable number of fleas, our swatter gets an upgrade: we can now extinguish all fleas in an interval of size 1 every second. Now is it possible to kill all the fleas? We think that it is.

However, to our disappointment, Cam confessed that he was lying about the universe being a line: the universe was, in fact, a plane. With every point on the plane having one flea, the humongous population of fleas seems to be impossible to fully swat. Whether these fleas will be extinguishable will remain as a problem for the readers.